Local excitations of a spin glass in a magnetic field

J. Lamarcq, J.-P. Bouchaud, and O. C. Martin[*]

Service de Physique de l'Etat Condensé, Orme des Merisiers — CEA Saclay, 91191 Gif sur Yvette Cedex France

We study the minimum energy clusters (MEC) above the ground state for the $3-d$ Edwards-Anderson Ising spin glass in a magnetic field. For fields B below 0.4, we find that the field has almost no effect on the excitations that we can probe, of volume $V \leq 64$. As found previously for $B = 0$, their energies decrease with V, and their magnetization remains very small (even slightly negative). For larger fields, both the MEC energy and magnetization grow with V, as expected in a paramagnetic phase. However, all results appear to scale as BV (instead of $B\sqrt{V}$ as expected from droplet arguments), suggesting that the spin glass phase is destroyed by any small field. Finally, the geometry of the MEC is completely insensitive to the field, giving further credence that they are lattice animals, in the presence or the absence of a field.

Introduction — Strong disorder or frustration in a system can lead to a low temperature phase with "frozen order". This is believed to be the case for spin glasses, the archetypes of such systems[1,2]. Does such a frozen phase persist under generic perturbations? Here we consider that question in the context of the Edwards-Anderson[3] Ising spin glass when the perturbation is an external magnetic field. The effect of the field is to align the spins in one direction and this bias may break up the spin glass ordering. Do arbitrarily small fields destroy the frozen order, or can the ordering co-exist with a field as long as it is not too large? This is a long-standing question that has been surprisingly difficult to answer.

From an experimental point of view, this question has been addressed only twice for systems of Ising spins. In the first study[4], the (non-equilibrium) properties were interpreted to suggest the presence of spin glass ordering at low fields. Several years later the same sample was re-analyzed and the lines of constant relaxation time determined[5]. From that, a scaling analysis was performed, leading to the conclusion that the relaxation time is finite for non-zero fields, and thus the system is paramagnetic. On the theoretical size, the situation remains very controversial. In the mean field picture, where one is guided by the Sherrington-Kirkpatrick[6] or other mean field models, the de Almeida-Thouless line[7] $B_{AT}(T)$ separates the paramagnetic and spin glass phases, and one has $B_{AT} > 0$ if $T < T_c$ (T_c is the critical temperature in zero field). Thus spin glass ordering continues in the presence of a sufficiently small magnetic field in the mean field picture. At present, there is no consensus whether this picture correctly describes what happens in the $d = 3$ Ising spin glass, nor even when d is large but finite. Indeed it has been argued by Fisher and Huse[8] that the magnetic field is always relevant, driving the system at large scales towards paramagnetism[9]. Then any non-zero magnetic field will destroy the spin glass ordering and will render the system paramagnetic beyond a length scale ℓ_B. In the droplet[8] or scaling[10] pictures, ℓ_B diverges as B vanishes like

$$\ell_B \propto B^{-2/(d-2\theta)} \qquad (1)$$

where θ is the spin stiffness exponent. Because of this divergence as B decreases, the relaxation times also diverge. The difficulty is thus to determine whether these relaxation times diverge only as $B \to 0$ or at some positive value $B_c > 0$. Such a question is nearly impossible to address from simulations because the time scales that can be reached always remain quite small, while equilibrium studies are confined to small systems for the same reason. In view of such hurdles, much numerical effort[11,12] in the last few years has focused instead on the zero temperature limit.

In this paper we also use zero temperature, but we study how minimum energy clusters[13] above the ground state are affected by a magnetic field. Essentially, we search for a putative critical field B_c; if there is an AT-line, B_c is then simply $B_{AT}(T = 0)$. Our investigation reduces to finding whether there is a phase transition in B at $T = 0$, and for that we need an order parameter. Since the field breaks the up-down symmetry, the Edwards-Anderson order parameter that gives the spatial average of the square magnetization at equilibrium is not of use. It is thus necessary to characterize the spin glass ordering otherwise, for instance by an infinite (spin glass) susceptibility or by the presence of irreversibility on all time scales. However these quantities are not accessible at $T = 0$, and so other measures of ordering are necessary. One characteristic of a spin glass order is sensitivity to external perturbations; one can thus see whether the ground state evolves chaotically with B up to a critical value B_c. The study in[11] showed that there was no sign of chaos when the lattice size was increased for values of $B \geq 1$, suggesting that $0 \leq B_c < 1.0$. This should be compared to the mean field value for connectivity 6 random graphs for which $B_{AT}(T = 0) \simeq 2.1$. Another study[12] considered how the ground state responded to a domain wall twist; this gave some evidence that the system had frozen order for $B \leq B_c \simeq 0.6$, although $B_c = 0$ could not be excluded. Our approach here probes the spin glass ordering through its *local* excitations; the energy to flip a cluster of spins behaves differently in a spin glass phase and in a paramagnetic phase, and we use this to test for spin glass ordering. First we shall

describe what these minimum energy clusters are; then we shall consider how their energies and magnetizations depend on the external magnetic field.

Minimum energy clusters — We start with the $3-d$ Edwards-Anderson (EA) model with periodic boundary conditions. The Hamiltonian is defined on a cubic lattice of $N = L^3$ spins,

$$H = - \sum_{<ij>} J_{ij} S_i S_j + B \sum_i S_i \ . \qquad (2)$$

The spins are Ising, i.e., $S_i = \pm 1$, and the nearest-neighbor interactions $\{J_{ij}\}$ are quenched random variables distributed according to a Gaussian law with zero mean and unit variance.

In this model, when B is set to 0, there is numerical evidence[14] of an ordered phase at low temperature where each spin i is frozen in a random direction $\langle S_i \rangle \neq 0$. We are interested in local excitations in the putative frozen phase, so following previous work[13], we define a Minimum Energy Cluster or MEC as the *connected* cluster of spins of lowest energy that contains a specified number of spins and a given, arbitrarily chosen site. The study of such MEC in *zero field* showed[13] that they are, perhaps surprisingly, fractals whose typical energy *decreases* with their size, in contrast with the properties expected for the Fisher-Huse droplets. This result is striking and we take it to be a signature of the spin glass phase, allowing us to probe now the case where $B > 0$.

Numerical method — Our measurements are performed on lattices with $L = 6$ and $L = 10$ with different values of the field taken in the range $0 \leq B \leq 1.5$; in all cases, we generated 100 disorder samples which was enough to obtain reasonably small statistical errors on our observables.

For each disorder sample, we first compute the ground state of the system using a genetic renormalization algorithm[15]. Then, we choose an arbitrary "reference" spin and flip it along with a cluster containing $V-1$ other spins connected to it. The goal is to find the MEC of size V, with the constraint that the reference spin is held flipped and the cluster is always connected. To find that MEC, we use non-local Kawasaki dynamics as in[13] and exchange Monte Carlo[16] with a set of 30 temperatures (between $T_1 = 0.5$ and $T_{30} = 3.0$). This search for the MEC is repeated five independent times to check whether the same cluster is found. (Since our algorithm is heuristic, we obtain an upper bound rather than the true energy of the MEC.) In practice, we find the MEC quite reliably for $V \leq 32$ as shown by our tests. Note that our optimization procedure has been improved since the analysis made in[13] in zero field; for instance, we have been able to find, for $V = 64$, lower excitations than before. With this improvement, we confirm our earlier conclusions but also we have been able to go to larger MEC sizes. In all that follows, we take $V_n = 2^n$, $1 \leq n \leq 6$; even though the true MEC are not always found at $V = 64$, we present the results also for that size as we believe the corresponding bias is small.

Geometry of MEC — Let us look at two geometric properties of MEC. Consider first the extension of MEC as a function of their volume V; our data for the mean end-to-end extensions are displayed in the insert of Fig. 1. Given these values, we conclude that MEC span the whole system when $V \approx 15$ if $L = 6$ and when $V \approx 30$ if $L = 10$.
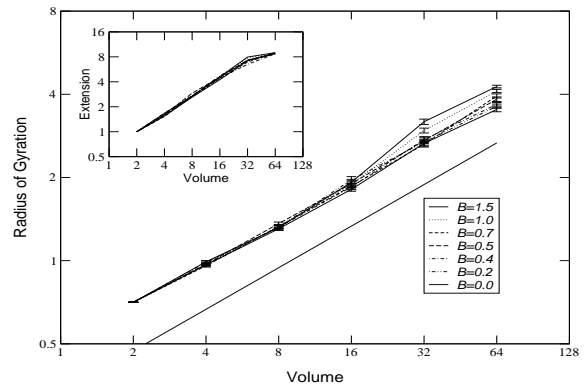

FIG. 1: Radius of gyration versus the volume of MEC at different values of the field B when $L = 10$. Insert: Mean end-to-end extension versus the volume. Note the log-log scale in both cases. The plain line has a slope $1/2$, corresponding to $d_f = 2$.

Second, consider the fractal dimension of the MEC. When $B = 0$, we argued[13] that configurational entropy is sufficient to drive MEC towards the lattice animal phase in which they are stringy objects with a fractal dimension $d_f = 2$. This excitation branch is thus distinct from the Fisher-Huse droplets which are compact objects with a fractal *boundary*. What happens in the presence of a field? It is useful to think of the large B limit which is strongly paramagnetic. The surface energy of a cluster is a random variable J_{ij} that is symmetric about zero, while its bulk energy is $\approx BV$. Clearly, energy minimization in a random environment will drive the clusters to extend their surface, leading to the lattice animal phase. We can see in Fig. 1 that this geometrical property, just like the extension, is nearly independent of the strength of the magnetic field. This shows that geometrical properties of MEC are *not* relevant to distinguish between a paramagnet and a spin glass, but it also gives further support to the claim that MEC are lattice animals even when $B = 0$.

Energies of MEC — Consider first the typical excitation energy $\overline{E}$ of a MEC, where $(\bar{\cdot})$ denotes the average over disorder. In the paramagnetic phase, i.e., for large B, $\overline{E}$ is expected to be self-averaging as $V \to \infty$. Since there is a non zero magnetization per spin, that energy should grow as $\overline{E(V)} \propto BV$, leading therefore to $\theta = 3$. Note however that because of the self-averaging property, the probability to find $E = O(1)$ should go to zero much faster than any power of V for large V; there are thus *no* low-energy excitations at large scales. Following an Imry-Ma argument, Fisher and Huse[8] argued that this should actually occur for any $B \neq 0$ for sufficiently large V's. The paramagnetic behavior should set in beyond a cross-over volume V_B, obtained by comparing the Zee-

man energy $B\sqrt{V}$ of a $B = 0$ droplet with its excitation energy $\Upsilon V^{\theta/d_f}$. The result is the cross-over volume $V_B \propto B^{-2d_f/(d_f-2\theta)}$, with the corresponding length being given in Eq. 1. Note that d_f is the (possibly fractal) dimension of the excitation. In the usual picture of compact droplets[17], one has $d_f = 3$ and $\theta \approx 0.2$. In the numerical investigation of MEC[13], it was found that the excitation clusters were instead fractal, $d_f \approx 2.0$, and that the effective θ was small and negative. Since θ is small compared to d_f in all cases, we expect roughly $V_B \propto (\Upsilon/B)^2$. Using the value $\Upsilon \approx 6$ from our numerical results, we obtain a rather large value $V_B \approx 36/B^2$.

Let us focus on the volume dependence of the mean MEC energy at different fields B. For all our B values, this mean initially *decreases* with volume; furthermore, this decay is compatible with a power law, $\overline{E(V)} \approx V^{\theta_f/d_f}$ with $\theta_f \approx -0.13$ as found in zero field[13]. From the point of view of the mean field picture, the behavior observed in our data is qualitatively as expected: below $B_c \approx 0.5$, the spin glass ordering leads to a decreasing $\overline{E(V)}$ at all V, while for $B > B_c$ we recover the paramagnetic behavior of an increasing energy when V is sufficiently large. Note however that the estimate $B_c \approx 0.5$ is far below the mean field value[11] associated with random graphs of connectivity 6, $B_{MF} \approx 2.1$. The important issue is therefore whether the curves eventually bend upwards at large V for all values of $B > 0$. In order to be more quantitative, we have attempted to scale the data, looking for a collapse when plotting $\overline{E(V)}/\overline{E(V_B)}$ as a function of V/V_B. Surprisingly, the data does not collapse well at all when $V_B \propto B^{-2}$ but merges much better if we take $V_B \propto B^{-1}$ as shown by the "scaling curve" in the insert of Fig. 2. This result suggests that even the low B curves might eventually bend upwards for large enough V; that could be interpreted as signaling the instability of the spin-glass phase in a field. We will return to this point later.
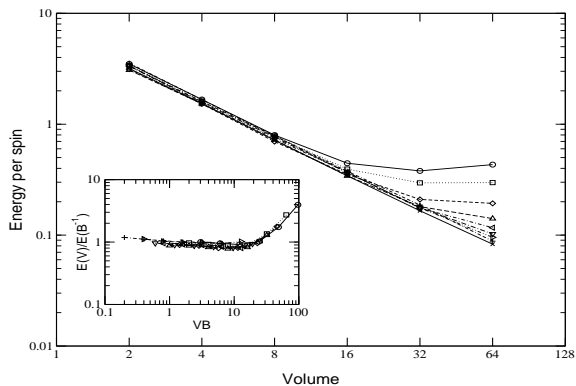

FIG. 2: Mean energy per spin versus volume at different values of B when $L = 10$. Insert: Scaling curve. $B = 1.5(\circ)$, $1.0(\square)$, $0.7(\diamond)$, $0.5(\triangle)$, $0.4(\triangleleft)$, $0.3(\triangledown)$, $0.2(\triangleright)$, $0.1(+)$, $0(\times)$

Note that the BV scaling obtained above is rather unexpected since it means that the influence of a small magnetic field on low energy excitations is actually *stronger* than anticipated by the Fisher-Huse argument. Although we do not have a clear understanding of this result, one

could slightly alter the argument as follows. For $B = 0$ the ground state has a zero magnetization per spin, which means, as recalled above, that the magnetization of a region of volume ℓ^3 is typically of order $\ell^{3/2}$. For $B \neq 0$, a non zero magnetization per spin χB (see Fig. 3) appears; this means that the spatial correlations *in the ground state* change for distances larger than ℓ_B such that $\ell_B^3 \chi B \approx \ell_B^{3/2}$, leading to $\ell_B \approx (\chi B)^{-2/3}$. Since the MEC are fractal, the excitations built at $B \neq 0$ are expected to be affected as soon as $V \sim \ell_B^{d_f}$, or, using $d_f = 2$, at $V \sim (\chi B)^{-4/3}$, a result that is closer to the scaling reported in Fig. 2.
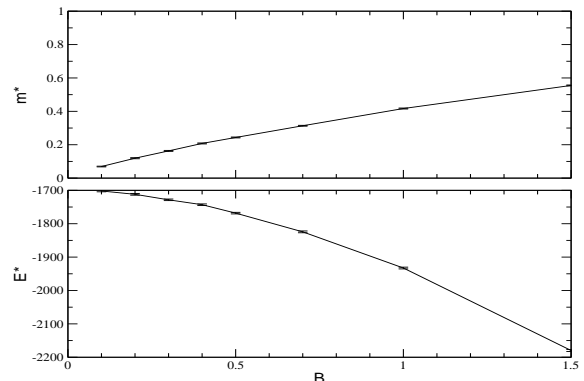

FIG. 3: Mean magnetization per site (upper panel) and mean energy (lower panel) of the ground state of a $N = 10^3$ spins Edwards-Anderson system, versus the magnetic field.

We have also studied the *distribution* of energies of MEC, for a given volume; the case $V = 32$ is given in Fig. 4. As expected, for large fields ($B = 1.0$ and 1.5), the distributions become symmetric around their maximum, and their weights for small excitation energies rapidly tend to zero. However for low fields, $B \leq 0.5$, we find that these distributions hardly change at all with B.
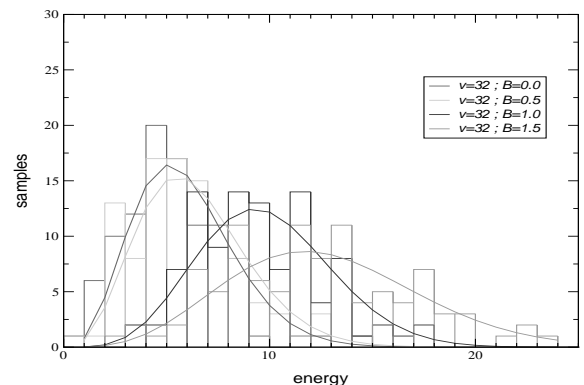

FIG. 4: Histogram of energies for $V = 32$ MEC, at different values of B. The lines are there to guide the eye.

Magnetization of MEC — Another observable of interest is the mean magnetization per site $m(V, B)$ of MEC of volume V in the presence of the magnetic field B. (The MEC's magnetization is defined *before* it is flipped.) In the droplet picture, the system is driven to the large B limit
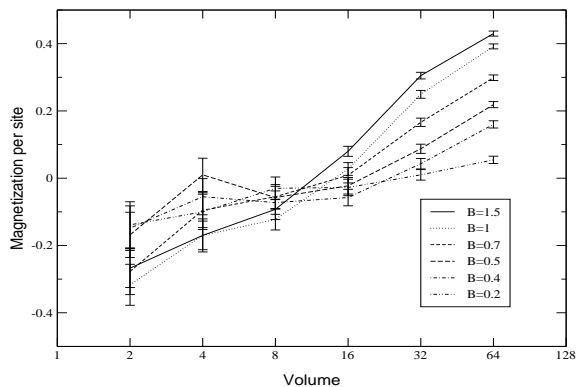
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FIG. 5: Mean magnetization per site versus V when $L = 10$, for different magnetic fields B.

when V grows as soon as $B \neq 0$. Thus, beyond the crossover scale V_B discussed above, $m(V, B)$ should converge to a function $m(B)$ that is non-zero for all $B \neq 0$. The situation is different in the mean field picture. Mean field predicts that the equilibrium magnetization is independent of the temperature T at low enough T. Now since the low T properties follow from those of the lowest-lying excitations, we see that these excitations cannot have a non-zero mean magnetization. The mean field picture then consists in having the large V limit of $m(V, B)$ vanish for $B \leq B_c$ and grow for $B > B_c$. In both pictures of the spin glass phase, $m(B)$ acts as an *order parameter* for the paramagnetic phase. Fig. 5 shows $m(V, B)$ as a function of V. We see that the magnetizations for $B \leq 0.5$ are indeed very small and in fact are negative rather than positive. Also, we see a clear upturn to positive m for $V = 32$, $B \gtrsim 0.4$; this upturn seems to be present even for smaller fields, suggesting that $B_c < 0.4$. This bound is more stringent than what we found from the *energies*

of the MEC. Finally, just like the energy, the magnetization appears to approximately rescale as a function of BV.

Conclusions — Our results show that when $B < 0.4$, the system behaves qualitatively just as in the case $B = 0$ (at least for the sizes studied here), whereas significant changes arise when $B \geq 0.4$. If there is a critical field, which appears to us rather unplausible, it must obey $B_c < 0.4$, a bound that is stronger than that most recently obtained[12]. This value is small compared to the value arising in the mean field approximation for which $B_c \approx 2.1$. Although we cannot completely rule out the mean field picture, the possibility of rescaling the different curves as a function of BV points towards the destruction of the spin-glass phase for $B \neq 0$. On the other hand, our data are also unexpected from the point of view of the droplet model. First, as emphasized before[13], MEC appear as fractal, rather than compact, objects. Second, the Fisher-Huse prediction of a crossover volume $V_B \propto B^{-2}$ appears not to be correct. We find that the magnetic field has a much stronger effect, since we obtain $V_B \propto B^{-1}$, which might be a consequence of the fractal nature of the MEC. Finally, the geometric properties of the MEC appear to be B independent, and most probably they are lattice animals for all B. Many of the points raised by the present study would be clarified if the corresponding results in $d = 4$ were available. Note that recent work[18] on Bethe lattices reveals a non zero value of the critical field B_c as expected.

* Laboratoire de Physique Théorique et Modèles Statistiques, Bâtiment 100 — Université Paris Sud, 91405 Orsay France

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